6 Characteristic Lengths and Times Associated with Processes in the Root Zone

P.A.C. Raats

Institute for Soil Fertility
Haren, the Netherlands

The Millers contributed to root ecology in two areas, which are:

1. Over a period of more than 30 yr, Bob Miller studied various aspects of the mechanical interaction between roots and soils;
2. Ed Miller was involved in two key developments concerning uptake of water by plant roots from soils.

Ed Miller supervised the dissertations of Wolf (1967) on uptake of water by growing root systems, and of Herkelrath (1975) and Herkelrath et al. (1977a, b) on the influence of soil water content and soil water potential upon uptake, leading to the "root contact" model. I will return to these studies in the course of this article.

To better understand poor root growth in some compacted soils, Gill and Miller (1956) studied the roles of mechanical impedance and oxygen supply. Inspiration for this study was derived from Pfeffer's late 19th century experiments on root growth pressures exerted by plants (Gill & Bolt, 1955). To minimize the roles of other factors, Gill and Miller used glass beads and tap water as a substrate, and sterilized, freshly germinated corn (*Zea mays* L.) seedlings as the plant material. They determined root growth as a function of the applied pressure, with the percent oxygen present in the soil atmosphere as a parameter.

The mechanics of root penetration was the subject of two other studies under Bob Miller's guidance. Evans (1965) developed a model for radial growth in a granular soil. This was followed by a study of radial resistance as a function of root size and spacing by Römkens and Miller (1971).

In recent years Bob Miller had occasion to combine his intermittent interest in plant roots and his continual interest in the physics of freezing

 Scaling in Soil Physics: Principles and Applications, SSSA Special Publication no. 25.

of soils in a study of frost upheaval of overwintering plants (Perfect et al., 1987, 1988). They found that two modes of upheaval can be distinguished. On the one hand, seedlings and transplants may be uprooted by surficial soil freezing in the fall and/or spring. On the other hand, well-anchored plants are displaced by deep frost penetration during midwinter.

Following some general comments on scaling Richards' Equation, the remainder of this article will deal with various aspects of the uptake of water, including the role of poor contact between root and soil and uptake by a growing root system.

SOME ASPECTS OF SCALING

To describe movement of water in unsaturated soils, nearly 60 yr ago Richards (1931) proposed the simplest possible balance of mass and balance of momentum, the latter expressed in terms of Darcy's Law. The balance of mass for the water may be written as

$$\delta\theta/\delta t = \underset{\sim}{\nabla}(\theta \underset{\sim}{v}) - u \qquad [1]$$

where t is the time, $\underset{\sim}{\nabla}$ is the vector differential operator, θ is the volumetric water content, $\underset{\sim}{v}$ is the velocity of the water, and u is the volumetric rate of uptake. The volumetric flux $\theta\underset{\sim}{v}$ is given by Darcy's Law:

$$\theta\underset{\sim}{v} = -k[h]\underset{\sim}{\nabla}h + k[h]\underset{\sim}{\nabla}z \qquad [2a]$$

$$= -D[\theta]\underset{\sim}{\nabla}\theta + k[\theta]\underset{\sim}{\nabla}z \qquad [2b]$$

$$= -\underset{\sim}{\nabla}\phi + k[\phi]\underset{\sim}{\nabla}z \qquad [2c]$$

where h is the tensiometer pressure head, z is a vertical coordinate with its origin at the soil surface and taken position downward, and the diffusivity D and the matric flux potential ϕ are defined by

$$D = kdh/d\theta \qquad [3]$$

$$\phi - \phi_0 = \int_{h_0}^{h} kdh = \int_{\theta_0}^{\theta} Dd\theta \qquad [4]$$

Symbols in brackets denote functional dependence. Unlike the dependence of k upon θ, the dependence of h upon θ is subject to hysteresis. As a consequence, Eq. [2b] and [2c] are, strictly, only valid for monotonic changes in water content from some initial condition with uniform θ and h.

The retention and conduction of water by soils are primarily governed by the relationships between h and θ, and between k and θ. These relationships vary widely among soils. The scaling theory of Miller and Miller (1956) is concerned with geometrically similar media characterized by length scales $\lambda_* = 1$ and λ. Figure 6–1 shows two geometrically similar media with geometrically similar distributions of water and of air. For such a pair, the STVF (surface tension, viscous flow)–theory implies that simple relationships exist between the pairs of water contents, pressure heads, and hydraulic conductivities:

1. Geometric similarity implies

$$\theta = \theta_* \qquad [5]$$

2. The inverse relationship between the pressure head and the mean radius of curvature implies

$$h = \lambda^{-1} h_* \qquad [6]$$

3. The linearized Navier-Stokes Equation at the microscopic scale implies that in Darcy's Law at the macroscopic scale the hydraulic conductivity satisfies

$$k = \lambda^2 k_* \qquad [7]$$

The three scaling rules just given are of the form (Raats, 1983)

$$v = \lambda^n v_* \qquad [8]$$

with integer n. Using the three primary scaling rules, secondary scaling rules can be inferred from Darcy's Law and from the volume balance for the water. Darcy's Law implies simple scaling rules for the spatial coordinates x, y, and z, the velocity $\underset{\sim}{v}$, the volumetric flux $\theta \underset{\sim}{v}$, the total head $H = h + z$, the diffusivity D, and the matric flux potential ϕ. The volume balance for the water implies scaling rules for the time t, and the volumetric

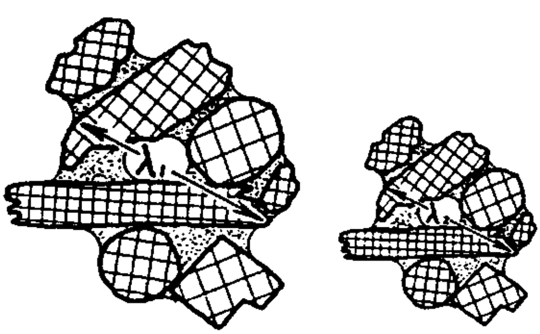

Fig. 6–1. Two geometrically similar media in similar states (Miller & Miller, 1956).

Table 6–1. Scaling rules and implied means and variances for a set of similar media with lognormally distributed length scales.

v	n	Mean of v	Variance of v
t	-3	-3μ	$9\sigma^2$
h	-1	$-\mu$	σ^2
H	-1	$-\mu$	σ^2
x, y, z	-1	$-\mu$	σ^2
θ	0	—	—
∇	1	μ	σ^2
$d\theta/dh$	1	μ	σ^2
D	1	μ	σ^2
ϕ	1	μ	σ^2
$k^{-1}\, dk/dh$ (= a for A in Table 6–2)	1	μ	σ^2
k	2	2μ	$4\sigma^2$
θv	2	2μ	$4\sigma^2$
v	2	2μ	$4\sigma^2$
$s = dk/d\theta$	2	2μ	$4\sigma^2$
dk/dh	3	3μ	$9\sigma^2$
u	3	3μ	$9\sigma^2$

rate of uptake u. Scaling rules for the water capacity $d\theta/dh$, the characteristic inverse length $a = k^{-1}dk/dh$, and the characteristic speed $s = dk/d\theta$, all three potentially a function of the water content, can also be inferred easily. Table 6–1 gives the values of n in Eq. [8] associated with the various parameters. Most noteworthy are the scaling rules for the spatial coordinates and the time.

1. The spatial coordinates and hence all macroscopic length scales in processes, should be inversely proportional to the microscopic length scale.
2. The time coordinate, and hence all time scales in processes, should be inversely proportional to the cube of the microscopic length scale.

An important implication of the power function dependence on λ^n of all the variables v in Table 6–1 is that if the length scale λ is lognormally distributed, then all the variables v/v_* will also be lognormally distributed (Raats, 1983). This is a consequence of the reproductive rule for lognormal distributions: if the variable χ is lognormally distributed with mean μ and variance σ then $e^a\chi^b$ is lognormally distributed with mean $a + b\mu$ and variance $(b\sigma)^2$.

The STVF theory of Miller and Miller concerns classes of geometrically similar media. An alternative method of defining classes of similar media is to describe the relationships among the water content θ, the pressure head h, and the hydraulic conductivity k for such classes. In effect this is often done in terms of parametric expressions for these relationships. Important examples are (see Table 6–2):

1. The Class of Mildly Nonlinear Soils with Linear $h[\theta]$ and Exponential $k[\theta]$ Relationships, Implying Exponential $D[\theta]$, $k[h]$, and $D[h]$ Relationships (Raats, 1983). The exponential $k[h]$ relationship linearizes

Table 6–2. Two classes of soils.

		Mildly nonlinear soils (A)	Power function soils (B)
Primary relationships	$h[\theta]$	$h_r + \gamma(\theta - \theta_r)$	$h_a(\theta/\theta_s)l$
	$k[\theta]$	$k_r \exp \beta(\theta - \theta_r)$	$k_a(\theta/\theta_s)^m$
Derived relationships	$D[\theta]$	$D_r \exp \beta(\theta - \theta_r)$ where $D_r = \gamma k_r$	$D_a(\theta/\theta_s)^n$ where $D_a = (lk_s h_a/\theta_s)$ $n = l + m - 1$
	$k[h]$	$k_r \exp a(h - h_r)$ where $a = \beta/\gamma$	$k_s(h/h_a)^p$ where $p = m/l$
	$D[h]$	$D_r \exp a(h - h_r)$	$D_s(h/h_a)^q$ where $q = n/l = (l + m - 1)/l$

the gravitational term in Darcy's Law expressed in terms of the matric flux potential and as such has been extremely useful in obtaining analytical solutions of steady flow problems, including problems involving uptake of water by plant roots (e.g., Raats, 1974a, 1976). Rereading Miller and Miller (1956), I noticed that they already pointed out that "$C(p)\{= a\}$ alone fully describes steady-flow behavior" and that "it may even be possible to approximate $C(p)\{= a\}$ by a constant for some purposes." For mildly nonlinear soils, the parameter a is proportional to the length scale λ.

2. The Class of Power Function Soils. As will be exemplified in this chapter, this class can in some cases also be used to obtain analytical solutions. The class of power function soils can be seen as a subclass of a superclass of soils, which shares flexibility with a rather sound basis in Poiseuillian flow in networks of capillaries (Raats, 1990). Members of this superclass are regularly used in numerical studies and as a basis for interpreting laboratory and field observations. For power function soils, the air entry pressure head h_a is inversely proportional to the length scale λ.

In the abstract for his lecture at Las Vegas, Ed Miller encourages "the use of the microlength λ as a natural part of any parameterized description of soil properties" (Miller, 1989). It may well be that this idea originated on a Northwest Orient flight Ed and I took sometime in 1969 (See Fig. 6–2).

SCALING OF UPTAKE

We have already seen that introducing the three basic scaling rules in the balance of mass shows that similarity requires that the volumetric rate of uptake is taken proportional to the cube of the length scale λ. This requirement is satisfied if the rooting depth is taken inversely proportional to λ and if the rate of transpiration is taken proportional to λ^2. The role of the rooting depth can be nicely demonstrated by considering the volumetric rate of uptake to be given by (Raats, 1974a, 1976)

$$u = f[z]T \tag{9}$$

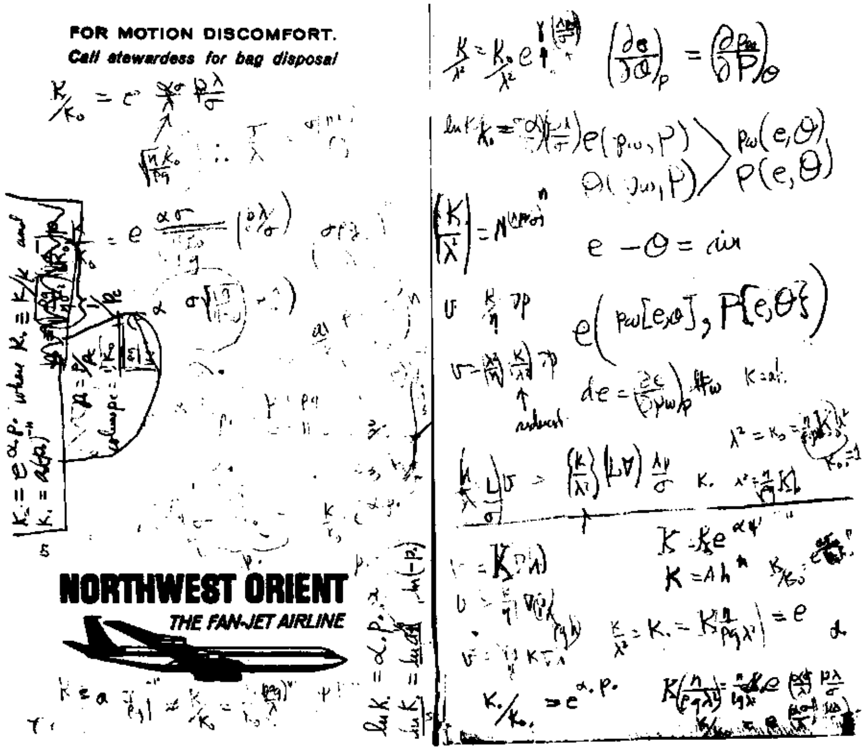

Fig. 6–2. Notes by Ed Miller, made sometime in 1969, on scaling of the exponential and power function dependencies of the hydraulic conductivity upon the soil water pressure. Note that, just as in Miller and Miller (1956), a reduced variable is denoted by a period following a symbol. Note further that, during the same flight, the equation of state for water in nonrigid soils was discussed briefly.

where T is the rate of transpiration and $f[z]$ is the distribution function for the uptake of water by plant roots. The distribution function

$$f = \delta^{-1} \exp(-z/\delta) \qquad [10]$$

where δ can be interpreted as an equivalent rooting depth, has turned out to be rather useful (Raats, 1974a, 1976). Together with the exponential $k[h]$ relationship for mildly nonlinear soils, this distribution function can be used to obtain solutions for steady flow involving uptake by plant roots. Specifically, for a constant rate of infiltration F_0 and in the absence of a water table, the distribution of the matric flux potential is given by (Raats, 1974a)

$$a\phi/F_0 = L + (1 - L)\frac{a\delta}{1 + a\delta}\exp(-z/\delta) \qquad [11]$$

where $L = (F_0 - T)/F_0$ is the leaching fraction. The dimensionless number

$a\delta$ embodies the interaction of the length scales of the soil and the root system. Taking a as the length scale of the soil, Eq. [11] can be written as

$$\phi_*/F_{0*} = L + (1 - L)\frac{\delta_*}{1 + \delta_*}\exp(-z_*/\delta_*) \quad [12]$$

In the example just given it is assumed that the plant has a certain demand for water and that this demand can be met at all times. This is certainly not always the case. In water balance models such as SWATRE, limited availability of water when the soil is either too wet or too dry is taken into account (cf., Feddes et al., 1978; Belmans et al., 1983).

Individual roots function at a meso-scale, which is intermediate between the microscopic Navier-Stokes scale and the macroscopic scale at which the uptake by plant roots is averaged over a large number of roots, as in Eq. [1]. Ever since the pioneering studies of Philip (1957) and Gardner (1960), mass balance Eq. [1] with u omitted and Darcy Eq. [2] have been used to analyze the movement of water in regions affected by individual roots. At this meso-scale of individual roots, even the simplest model of uniformly distributed parallel roots requires two length scales, r_0, the radius of the root, and r_1, the outer radius of the hollow cylinder of soil associated with the root. With flow to individual roots are also associated two characteristic times, t_d and $t_{s/d}$ defined by

$$t_d = r_1^2/\overline{D}, \qquad t_{s/d} = (1 - \rho_0^2)\theta_i B/T \quad [13]$$

where $\overline{D}$ is the mean of the soil water diffusivity in the appropriate range, θ_i is the initial water content, and B is the rooting depth. The time t_d characterizes the diffusive transport of the water to the root. The time $t_{s/d}$ arises from the ratio of the supply $(1 - \rho_0^2)\theta_i$ of water in the soil and the demand T/B by the plant, where $\rho_0 = r_0/r_1$.

To describe the flow to an individual root, it is convenient to introduce the dimensionless radial coordinate ρ, time τ, soil water depletion Δ, and diffusivity $\mathfrak{D}$:

$$\rho = r/r_1 \qquad \tau = t/t_d \quad [14]$$

$$\Delta = (\theta_i - \theta)/\theta_i \qquad \mathfrak{D} = D/\overline{D} \quad [15]$$

It turns out that the length scale r_0 and the time scale $t_{d/s}$ occur in the flow problem through the dimensionless parameters ρ_0 and $\tau_{s/d}$ defined by

$$\rho_0 = r_0/r_1 \qquad \tau_{s/d} = t_{s/d}/t_d \quad [16]$$

In terms of the dimensionless variables the uptake problem can be stated as in Table 6–3. Up to the dimensionless time τ_{crit}, the solution of the flow problem depends on the soil property $\mathfrak{D}[\Delta]$ and the parameters ρ_0

Table 6–3. Uptake of water by a plant root in terms of dimensionless variables.

$\dfrac{\delta\Delta}{\delta\tau} = \dfrac{\delta}{\delta\rho}\,\mathcal{D}\,\dfrac{\delta\Delta}{\delta\rho}$		
$\tau = 0$	$\rho_0 < \rho < 1$	$\Delta = 0$
$\tau > 0$	$\rho = 1$	$\dfrac{\delta\Delta}{\delta\rho} = 0$
A. Constant rate of uptake		
$0 < \tau < \tau_{crit}$	$\rho = \rho_0$	$\mathcal{D}\dfrac{\delta\Delta}{\delta\rho} = \dfrac{1-\rho_0^2}{2\rho_0}\;\dfrac{t_d}{t_{s/d}}$
B. Falling rate of uptake		
$\tau > \tau_{crit}$	$\rho = \rho_0$	$\Delta \rightarrow \Delta_{lim}$

and $\tau_{s/d}$. In particular this means that τ_{crit} will depend on $\mathcal{D}[\Delta]$, ρ_0, and $\tau_{s/d}$. The evaluation of τ_{crit} is the central point of interest in the analysis of uptake by plant roots in the Ph.D. thesis of de Willigen and van Noordwijk (1987). For $\tau > \tau_{crit}$ the solution may eventually be governed mainly by the ability of the soil to supply water to the soil/root interface.

Stating the flow problem in terms of dimensionless variables greatly increases the efficiency of computations, because the six variables θ_i, D_1, r_0, r_1, T, and B have been coalesced into the three variables $\mathcal{D}_0$, ρ_0, and $\tau_{s/d}$. Further efficiency is obtained by considering Miller scaling. Any solution of the uptake problem stated in Table 6–3 applies to any Miller similar flow. Similarity requires that the length scales r_0, r_1, and B are proportional to λ^{-1}, that the rate of transpiration is proportional to λ^2, and that the time scales t_d and $t_{s/d}$ are proportional to λ^{-3}. Therefore, the coarser the soil, the thinner and more closely spaced the individual roots should be, the smaller the rooting depth should be, and the more rapid the flow process should evolve. Horticulturalists create a wide range of root environments, which tend to being Miller similar: they use coarse substrates, in thin layers, inhabited by dense root systems, being irrigated frequently.

LIMITED CONTACT BETWEEN ROOT AND SOIL

Figure 6–3 shows schematically the poor contact between root and soil. Herkelrath et al. (1977b) suggested that as long as the potential transpiration can be met, the transport from the soil to the xylem is described by

$$0 < t < t_{crit}\,,\; R = R_0,\; D\,\frac{\delta\theta}{\delta r} = \frac{r_1^2}{2r_0\delta}\,E_{pot} = C\,\frac{\theta_0}{\theta_s}\,\{h[\theta_0] - h_{xylem}\} \qquad [17]$$

where C is the conductance of the region between the soil and the xylem. The degree of saturation θ_0/θ_s of the soil at the soil/root interface is a

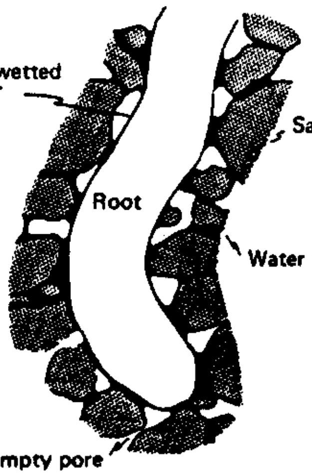

Fig. 6–3. Limited contact between root and soil (Herkelrath et al., 1977b).

factor accounting for the poor contact. From Eq. [17] it follows that at time t_{crit}

$$h[\theta_{0\text{crit}}] = h_{\text{xylem crit}} + \frac{r_1^2}{2r_0\delta}\frac{\theta_s}{\theta_{0\text{crit}}}\frac{E_{\text{pot}}}{C} \qquad [18]$$

or in scaled form

$$h_*[\theta_{*0\text{crit}}] = h_{*\text{xylem crit}} + \frac{r_{*1}^2}{2r_{*0}\delta_*}\frac{\theta_{*s}}{\theta_{*0\text{crit}}}\frac{E_{*\text{pot}}}{C_*} \qquad [19]$$

where

$$C = \lambda^3 C_* \qquad [20]$$

This means that Miller scaling requires that the coarser the soil, roots should become not only thinner and more closely spaced, but their cortex should become more permeable.

For given values of $h_{*\text{xylem crit}}$, r_{*1}, r_{*0}, θ_{*s}, $E_{*\text{pot}}$, and C_*, Eq. [19] relates the pressure head $h_{*\text{crit}}$ to the water content $\theta_{*0\text{crit}}$ of the soil at the soil/root interface. The infinity of pairs ($h_{0\text{crit}}$, $\theta_{0\text{crit}}$) or ($h_{*0\text{crit}}$, $\theta_{*0\text{crit}}$) is reduced to the single pair by determining the intersection of Eq. [18] or [19] with the soil water retention curve. For ($h_{0\text{crit}}$, $\theta_{0\text{crit}}$) this was done graphically by van Noordwijk (1983; see also de Willigen & van Noordwijk, 1987). A related graphical technique with the role of the retention curve replaced by the relationship between the hydraulic conductivity and the pressure head was used earlier in an analysis of steady infiltration into crusted soils (Raats, 1974b). Except for the presence of θ_0/θ_s, the "root contact model" is analoguous to the model commonly used to describe flow across a crust.

An alternative approach to modeling limited soil/root contact is to consider flow in a plane perpendicular to a root, assuming that the circumference consists of two parts, one part in contact with wet soil, another part in contact with air (de Willigen & van Noordwijk, 1987). The availability of the water is then reduced by the change from a purely radial flow pattern to a pattern in which also angular components of the flux are involved. The smaller the root/soil contact, the smaller the fraction of the potentially available water that can be acquired at a certain rate.

Some consequences of partial soil/root contact have been analyzed by de Willigen and van Noordwijk (1987) not only for uptake of water, but also for uptake of plant nutrients and for exchange of gases with the soil atmosphere. Although partial contact reduces the availability of water and nutrients, it enhances the exchange of gases between roots and soil atmosphere. In structured soils, roots have a tendency to follow macropores in the form of cracks, worm holes, and holes left behind by decayed roots. To what a degree partial contact is a consequence of the inability of roots to penetrate or an innate strategy assuring proper future functioning may be difficult to determine. Clustering is another feature of root distributions, especially in structured soils, limiting availability of water and nutrients (de Willigen & van Noordwijk, 1987).

UPTAKE OF WATER BY A GROWING ROOT SYSTEM

Wolf (1967) analyzed the uptake of water at a root front in an infinite, uniform soil. He discussed three cases: (i) transient flow to a stationary root front, (ii) steady flow to a moving root front, and (iii) transient flow to a moving root front. In the following, some aspects of this problem will be discussed.

Assume that the entire water uptake occurs at a plane densely populated with root tips, moving at a velocity v_f, and that the water moves in the direction z perpendicular to this plane. To discuss this class of flows, it is convenient to introduce a coordinate frame of reference that moves with the root front. Equation [21] defines the moving coordinate Z in terms of the stationary coordinate z, the time t, and the velocity of the root front v_f.

$$Z = z - v_f t \qquad [21]$$

The corresponding transformations of the space and time derivatives are

$$\frac{\delta\cdot}{\delta Z} = \frac{\delta\cdot}{\delta z}, \qquad \frac{\delta\cdot}{\delta t\big|_Z} = \frac{\delta\cdot}{\delta t\big|_z} + v_f \frac{\delta\cdot}{\delta z} \qquad [22]$$

In terms of Z and t, Eq. [1] becomes

$$\frac{\delta\theta}{\delta t|_Z} = \frac{\delta}{\delta Z}\,\theta(v - v_f) \qquad [23]$$

Assuming that behind the root front the velocity of the water is zero, the mass balance at the root front reduces to (cf., Raats, 1972)

$$\theta^+(v^+ - v_f) = -\hat{u} - \theta^- v_f \qquad [24]$$

On the left-hand side of Eq. [24] appears the flux of water relative to the root front. On the right-hand side of Eq. [24], the first term represents the rate of uptake of water per unit area, and the second term represents the flux of water into the zone behind the root front.

In terms of Z and t, Eq. [2] becomes

$$\theta v = -D\,\frac{\delta\theta}{\delta Z} \qquad [25]$$

where the gravitational force acting on the water has been neglected. At the root front the pressure head and therefore also the water content will be continuous

$$\theta^+ = \theta^- = \theta_0 \qquad [26]$$

Introducing Eq. [26] in Eq. [24] gives

$$\theta^+ v^+ = -\hat{u} \qquad [27]$$

Equations [23] and [25], together with the initial condition $\theta[Z, t] = \theta_i$ and the boundary conditions, Eq. [26] and [27], describe the flow to a root front.

Transient flow to a stationary root front can be treated by means of the so-called Boltzmann transformation. The cumulative uptake increases as $t^{1/2}$, the rate of uptake decreases as $t^{-1/2}$. The details of the pressure head and water content distributions at successive times depend on the physical properties of the soil.

With a stationary, plane root front the flow does not tend to become steady. If the root front is moving, however, the flow does tend to become steady in the frame of reference moving with the root front. When this happens, Eq. [23] reduces to

$$\frac{\delta}{\delta Z}\,\theta(v - v_f) = 0 \qquad [28]$$

Integration of Eq. [28] gives

$$\theta\,(v - v_f) = c \qquad [29]$$

Because for $Z \to \infty$, $\theta \to \theta_i$ and $v \to 0$

$$c = -\theta_i v_f \quad [30]$$

From Eq. [25], [29], and [30] it follows that

$$-D\frac{\delta\theta}{\delta Z} = -(\theta_i - \theta)v_f \quad [31]$$

On the left-hand side of Eq. [31] appears the flux at any Z. This flux ranges from its maximum value $-(\theta_i - \theta_0)v_f$ at $Z = 0$ to zero at $Z \to \infty$.

Integration of Eq. [31] gives

$$\frac{Zv_f}{D_0} = -\int_{\ln(\theta_i - \theta_0)}^{\ln(\theta_i - \theta)} (D/D_0)d\ln(\theta_i - \theta) \quad [32]$$

To integrate Eq. [32], D must be known as a function of ln $(\theta_i - \theta)$. Following are closed solutions for three $D[\theta]$ functions

1. Linear soil with $D = D_0$

$$\frac{Zv_f}{D_0} = -\ln\frac{\theta_i - \theta}{\theta_i - \theta_0} \quad [33]$$

According to Eq. [33] the dimensionless water content $(\theta_i - \theta)/(\theta_i - \theta_0)$ is an exponential function of the dimensionless distance Zv_f/D_0. This solution was also given by Wolf (1967).

2. Mildly nonlinear soils (see Table 6–2)

$$\frac{Zv_f}{D_0} = \exp\beta(\theta_i - \theta_0)\{(-E_i[-\beta(\theta_i - \theta)]) - (-E_i[-\beta(\theta_i - \theta_0)])\} \quad [34]$$

This solution is new.

3. Power function soils (see Table 6–2)

$$\frac{Zv_f}{D_0} = -\left(\ln\frac{\theta_i - \theta}{\theta_i - \theta_0} + \sum_{n=1}^{p}\frac{1}{n}\frac{\theta^n - \theta_0^n}{\theta_i^n}\right) \quad [35]$$

For $p = 0$, equation reduces to Eq. [33]. For $p = 1$, 2, and 4, Eq. [35] reduces to equations given by Wolf (1967). Figure 6–4 shows observed and calculated distributions of water content behind and ahead of a root front.

A quantity of particular interest is the depletion W ahead of moving root front

$$W = \int_0^\infty(\theta_i - \theta)dZ = \int_0^\infty\frac{D}{v_f}\frac{d\theta}{dZ}dZ$$

$$= \int_{\theta_0}^{\theta_i} Dd\theta/v_f = \int_{h_0}^{h_i} kdh/v_f = \frac{\phi_i - \phi_0}{v_f} \quad [36]$$

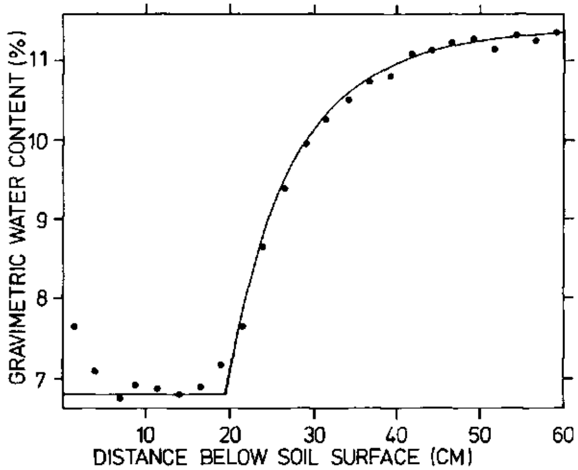

Fig. 6–4. Observed (data points) and calculated (curve) distribution of water content behind and ahead of a root front located at 20 cm below the soil surface (adapted from Wolf, 1967).

where Eq. [31] has been used. The simple evaluations of W for linear, mildly nonlinear, and power function soils are left to the reader. Equation [36] states that the effectiveness of the matric flux potential difference $\phi_i - \phi_0$ for delivering water to the root front from the region not yet explored by the root system is inversely proportional to the velocity v_f of the root front.

CONCLUDING REMARKS

Thirty years ago, when I was a M.Sc. student at Wageningen Agricultural University, Gerry Bolt asked me to determine and explain water retention curves of mixtures of sand and montmorillonite. This called for study of two, then recent developments in soil science: double layer theory for clays and STVF-theory for sands (and glass beads). I am grateful that at such an early stage I was introduced to those two far-reaching physical-mathematical models and learned about their limitations. Despite the fact that I hardly added to what Bolt and Miller (1958) had already written, theories of such calibre became a lasting interest.

REFERENCES

Belmans, C., J.G. Wesseling, and R.A. Feddes. 1983. Simulation model of the water balance of a cropped field: SWATRE. J. Hydrol. 63:271–286.

Bolt, G.H., and R.D. Miller. 1958. Calculation of total and component potentials of water in soil. Trans. Am. Geophys. Union 39:917–928.

de Willigen, P., and M. van Noordwijk. 1987. Roots, plant production and nutrient use efficiency. Ph.D. thesis. Agricultural Univ., Wageningen, the Netherlands.

Evans, G.N. 1965. A model of radial root growth in a granular soil Ph.D. thesis. Cornell Univ. (Diss. Abstr. 26:2943).

Feddes, R.A., P.J. Kowalik, and H. Zaradny. 1978. Simulation of field water use and crop yield. PUDOC, Wageningen, the Netherlands.

Gardner, W.R. 1960. Dynamic aspects of water availability to plants. Soil Sci. 89:63–73.

Gill, W.R., and G.H. Bolt. 1955. Pfeffer's studies of the root growth pressures exerted by plants. Agron. J. 47:166–168.

Gill, W.R., and R.D. Miller. 1956. A method for study of the influence of mechanical impedance and aeration on the growth of seedling roots. Soil Sci. Soc. Am. Proc. 20:154–157.

Herkelrath, W.N. 1975. Water uptake by plant roots. Ph.D. thesis. Univ. of Wisconsin (Diss. Abstr. 36:3710).

Herkelrath, W.N., E.E. Miller, and W.R. Gardner. 1977a. Water uptake by plants: I. Divided root experiments. Soil Sci. Soc. Am. J. 41:1033–1038.

Herkelrath, W.N., E.E. Miller, and W.R. Gardner. 1977b. Water uptake by plants. II. The root contact model. Soil Sci. Soc. Am. Proc. 41:1039–1043.

Miller, E.E. 1989. Perspectives on scaling in soil physics. p. 189–190. *In* Agronomy abstracts. ASA, CSSA, and SSSA, Madison, WI.

Miller, E.E., and R.D. Miller. 1956. Physical theory for capillary flow phenomena. J. Appl. Phys. 27:324–332.

Perfect, E., R.D. Miller, and B. Burton. 1987. Root morphology and vigor effects on winter heaving of established alfalfa. Agron. J. 79:1061–1067.

Perfect, E., R.D. Miller, and B. Burton. 1988. Frost upheaval of overwintering plants: A quantitative field study of the displacement process. Arct. Alp. Res. 20:70–75.

Philip, J.R. 1957. The physical principles of soil water movement during the irrigation cycle. p. 8.125–8.154. *In* Commission on Irrigation and Drainage Third Congress. Int. Comm. on Irrig. and Drainage, San Francisco, CA.

Raats, P.A.C. 1972. Jump conditions in the hydrodynamics of porous media. *In* Fundamentals of Transport Phenomena in Porous Media, IAHR-ISSS Symp. at Guelph, Ontario 1:155–173. 7–11 August. Univ. of Guelph, Guelph, ON, Canada.

Raats, P.A.C. 1974a. Steady flows of water and salts in uniform soil profiles with plant roots. Soil Sci. Soc. Am. Proc. 38:717–722.

Raats, P.A.C. 1974b. Steady infiltration into crusted soils. 10th Int. Congress of Soil Sci. (Moscow) Trans. 1:75–80.

Raats, P.A.C. 1976. Analytical solutions of a simplified flow equation. Trans. ASAE 19:683–689.

Raats, P.A.C. 1983. Implications of some analytical solutions for drainage of soil water. Agric. Water Manage. 6:161–175.

Raats, P.A.C. 1990. A superclass of soils. Proceedings of the International workshop on "Indirect methods for estimating the hydraulic properties of unsaturated soils," Riverside, CA, 11–13 Oct. 1989. USDA-ARS, U.S. Salinity Lab., Riverside, CA.

Richards, L.A. 1931. Capillary conduction of liquids through porous mediums. Physics 1:318–333.

Römkens, M.J.M., and R.D. Miller. 1971. Predicting root size and frequency from one-dimensional consolidation data—A mathematical model. Plant Soil 35:237–248.

van Noordwijk, M. 1983. Functional interpretation of root densities in the field for nutrient and water uptake. p. 207–226. *In* Wurzelökologie und ihre Nutzanwendung/Root Ecology and its Practical Application. Int. Symp. Gumpenstein, 1982, Bundesanstalt Gumpenstein, A-8952 Irdning. Published by Bundesanstallt für alpenländische Landwirtschaft, A–8952 Irdning.

Wolf, J. 1967. The role of root growth in supplying moisture to plants. Ph.D. thesis. The Univ. of Rochester, New York (Diss. Abstr. 68–15869).